

Non-perturbative $O(a)$ improvement of the vector current *M. Guagnelli^a and R. Sommer^b^aCERN, Theory Division, 1211 Geneva 23, Switzerland^bDESY-IFH, Platanenallee 6, D-15738 Zeuthen, Germany

We discuss non-perturbative improvement of the vector current, using the Schrödinger Functional formalism. By considering a suitable Ward identity, we compute the improvement coefficient which gives the $O(a)$ mixing of the tensor current with the vector current.

1. INTRODUCTION

It is well known that in Wilson's lattice QCD discretization errors receive contributions that are linear in the lattice spacing. The most obvious recipe to reduce them — namely to go to the continuum limit by performing numerical simulations at smaller lattice spacings — represents still a hard task, even in the quenched approximation.

Following Symanzik [1], lattice artifacts can be removed order by order in a by adding appropriate higher-dimensional operators to the action and to the fields whose correlation functions are of interest. If one restricts oneself by requiring improvement only for on-shell quantities [2], such as particle masses and matrix elements between physical states, the structure of the improved action for QCD and of the improved currents is rather simple. The general theory was developed in detail in ref. [3]. In the following we will use the same notation without further reference.

In the quenched approximation, the improved action was determined non-perturbatively for couplings $0 \leq g_0 \leq 1$ [4].

The renormalized and improved axial-vector and vector currents are parametrized as

$$\begin{aligned} (A_R)_\mu^a &= Z_A(1 + b_A am_q)(A_I)_\mu^a, \\ (V_R)_\mu^a &= Z_V(1 + b_V am_q)(V_I)_\mu^a, \end{aligned}$$

with

$$\begin{aligned} (A_I)_\mu^a &= A_\mu^a + ac_A \tilde{\partial}_\mu P^a, \quad \tilde{\partial}_\mu = \frac{1}{2}(\partial_\mu + \partial_\mu^*), \\ (V_I)_\mu^a &= V_\mu^a + ac_V \tilde{\partial}_\nu T_{\mu\nu}^a. \end{aligned}$$

In the above expressions, the coefficients c_X and b_X are functions of the bare coupling, and

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are known to 1-loop order of perturbation theory[5,6]. Moreover, the normalizations (Z_A and Z_V) and the improvement coefficients c_A and b_V have been determined non-perturbatively in the quenched approximation for $g_0 \leq 1$ [4,7]. As a next step, we determine c_V , thus completing the knowledge of the improved vector current. The computation of other improvement coefficients is discussed in [9].

Knowledge of c_V is, for example, required for the computation of vector meson decay constants. The relative contribution of $a\tilde{\partial}_\nu T_{\mu\nu}$ to these decay constants can be, at $g_0^2 = 1$, as large as $0.3 \times c_V$ [8]. Although the perturbative estimate [6],

$$c_V = -0.01225(1) \times g_0^2 C_F + O(g_0^4), \quad C_F = 4/3, (1)$$

suggests an effect of less than 1% on the decay constants, our preliminary non-perturbative results determine c_V to be much larger in magnitude for $g_0^2 \simeq 1$.

2. THE STRATEGY

Chiral Ward identities relate correlation functions of axial-vector and vector currents. In the $O(a)$ improved theory and for zero quark mass these identities can be written in a form which is valid up to error terms that are quadratic in a . Since the $O(a)$ -improved axial-vector current as well as Z_V and Z_A are known, one can use a particular Ward identity in which the only unknown is c_V .

Our starting point is the Ward identity (in the continuum):

$$\int_{\partial R} d\sigma_\mu(x) \epsilon^{abc} \langle A_\mu^a(x) A_\nu^b(y) Q^c \rangle - \quad (2)$$

$$2m \int_R d^4x \epsilon^{abc} \langle P^a(x) A_\nu^b(y) \mathcal{Q}^c \rangle = 2i \langle V_\nu^c(y) \mathcal{Q}^c \rangle,$$

where the space-time region R with boundary ∂R contains the point y and \mathcal{Q}^c is a source located outside R .

We then impose Schrödinger Functional boundary conditions [10], choose $\nu = k$ and specify the source,

$$\mathcal{Q}_k^c = a^6 \sum_{\mathbf{y}, \mathbf{z}} \bar{\zeta}(\mathbf{y}) \gamma_k \frac{\tau^c}{2} \zeta(\mathbf{z}), \quad (3)$$

where $\bar{\zeta}, \zeta$ are the quark fields at the boundary $x_0 = 0$ [3]. Finally the region R is specified to be the space-time volume between the hyperplanes $x_0 = t_1$ and $x_0 = t_2$.

On a lattice with finite spacing a , we then expect that

$$a^3 \sum_{\mathbf{x}} \epsilon^{abc} \langle [(A_R)_0^a(t_2, \mathbf{x}) - (A_R)_0^a(t_1, \mathbf{x})] \times (A_R)_k^b(y) \mathcal{Q}_k^c \rangle = 2i \langle (V_R)_k^c(y) \mathcal{Q}_k^c \rangle + \mathcal{O}(a^2), \quad t_1 < y_0 < t_2, \quad (4)$$

is valid with correction terms of order a^2 .

Note that passing from eq. (2) to eq. (4) we have specialized to $m = 0$. The reason for this is as follows. For finite quark mass, the second term on the left hand side of eq. (2) contains zero separations $x - y$. Since it is not an integral over an on-shell correlation function, it may have lattice artifacts of order a and cannot be used to formulate an improvement condition.

In terms of the bare unimproved correlation functions,

$$k_V(x_0) = -\frac{1}{9} \langle V_k^a(x) \mathcal{Q}_k^a \rangle, \quad (5)$$

$$k_T(x_0) = -\frac{1}{9} \langle T_{k0}^a(x) \mathcal{Q}_k^a \rangle, \quad (6)$$

as well as the bare improved correlation function,

$$k_{AA}^I(x_0, y_0) = \frac{i}{18} \sum_{\mathbf{x}} \epsilon^{abc} \langle (A_I)_0^a(x) (A_I)_k^b(y) \mathcal{Q}_k^c \rangle, \quad (7)$$

eq. (4) may be rewritten, for $t_1 < x_0 < t_2$, as

$$Z_V[k_V(x_0) + ac_V \tilde{\partial}_0 k_T(x_0)] = Z_A^2[k_{AA}^I(t_2, x_0) - k_{AA}^I(t_1, x_0)] + \mathcal{O}(a^2). \quad (8)$$

At zero quark mass m , this equation can be solved for c_V . However, due to a peculiarity of the

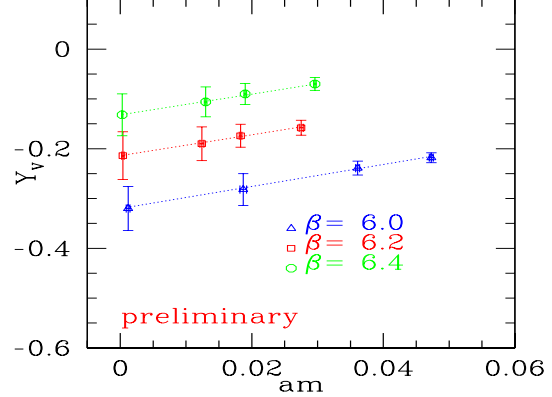


Figure 1. Dependence of Y_V on the quark mass.

quenched approximation, *i.e.* the appearance of zero-modes in the quark propagator, it is not always possible to simulate directly at zero quark mass [4]. For the particular boundary conditions and lattice size employed, here, this phenomenon is relevant when $\beta = 6/g_0^2 \leq 6.4$. In this region we have to perform an extrapolation to zero quark mass. However, in the course of the numerical simulations it turned out that c_V , implicitly defined by eq. (8) also for finite m , is a steep function of m and could not be reliably extrapolated to zero quark mass when we were not able to simulate at very small masses. The reason for this strong dependence is related to the fact that in the Ward identity (4) the mass term was left out: as a consequence, eq. (8) is not even valid in the continuum limit.

It is therefore natural to repeat the calculation, keeping the mass term. Eq. (4) is then modified by an additional term and has corrections of order $\mathcal{O}(am)$ instead of $\mathcal{O}(m)$. We solve the resulting equation formally for c_V and denote the solution at finite mass by Y_V , which coincides with c_V as $m \rightarrow 0$. As expected, this limit is now reached very smoothly (see figure 1).

In a non-perturbative calculation, c_V depends on the choices made for the various kinematical parameters such as T/L or the particular Schrödinger Functional background field. Different choices lead to a variation of c_V itself that is $\mathcal{O}(a)$. This is unavoidable and one must make a

specific choice; other choices would merely change the size of the $O(a^2)$ effects after improvement. On the other hand, one should search for values of the kinematical parameters such that the $O(a^2)$ terms are small compared to the improvement term $a\tilde{\partial}_0 k_T(x_0)$. To achieve this, we first of all choose t_1 and t_2 in such a way that the time separations between the various fields in the correlation functions are as large as possible:

$$t_1 = T/4, \quad t_2 = 3T/4. \quad (9)$$

To fix the other parameters, we studied and tried to minimize the cutoff effects in eq. (8) at tree level of perturbation theory (the interested reader will find details in ref. [11]).

With our final choices, we observe reasonably weak cutoff effects at tree level. Apart from this perturbative study, we also performed a rough numerical investigation at the largest values of the gauge coupling considering different improvement conditions (*i.e.* different choices of L/a and other parameters), and observed the extracted values for c_V to be the same within errors. The final definition of c_V includes an average over the three central time slices, x_0 , which reduces statistical uncertainties.

3. RESULTS

Our preliminary results are summarized in figure 2. Here we show the non-perturbatively computed values of c_V as a function of the bare coupling g_0^2 . The errors on c_V receive a significant contribution from the (statistically independent) error on the ratio Z_A^2/Z_V .

In the region $\beta \leq 6.4$ we observe sizeable values for c_V , far from 1-loop perturbation theory. In particular, at $g_0 = 1$ the effect of the improvement term on the value of the vector meson decay constant can be as large as 10%. When g_0^2 approaches 0, we obtain agreement with the perturbative result.

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REFERENCES

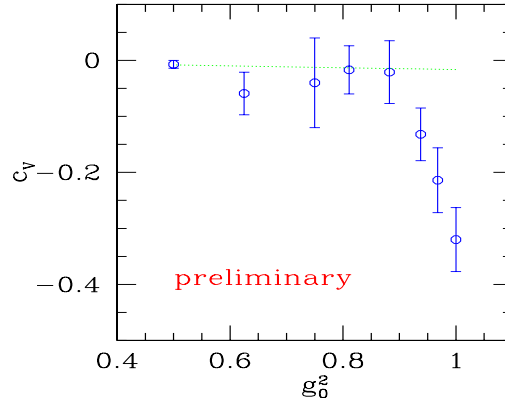


Figure 2. c_V versus g_0^2 . The dotted line is the 1-loop perturbative result.

1. K. Symanzik, Nucl. Phys. B226 (1983) 187 and 205.
2. M. Lüscher and P. Weisz, Commun. Math. Phys. 97 (1985) 59; Commun. Math. Phys. 98 (1985) 433
3. M. Lüscher, S. Sint, R. Sommer and P. Weisz, Nucl. Phys. B478 (1996) 365.
4. M. Lüscher, S. Sint, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. B491 (1997) 323.
5. M. Lüscher and P. Weisz, Nucl. Phys. B479 (1996) 429.
6. S. Sint and P. Weisz, hep-lat/9704001
7. M. Lüscher, S. Sint, R. Sommer and H. Wittig, Nucl. Phys. B491 (1997) 344.
8. M. Göckeler *et al.*, hep-lat/9707021.
9. G. de Divitiis, talk at this conference.
10. M. Lüscher, R. Narayanan, P. Weisz and U. Wolff, Nucl. Phys. B384 (1992) 168; S. Sint, Nucl. Phys. B421 (1994) 135, Nucl. Phys. B451 (1995) 416.
11. M. Guagnelli and R. Sommer, work in progress.